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Phase Transition in Extended Electroweak Theory.

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ABSTRACT

The phase transition in the Weinberg-Salam-Glashow (GSW) electroweak theory extended by the majoron and dilaton field is considered. The possibility of the boson condensation in the extreme conditions in the standard electroweak theory is shown. The first order phase transition induces by the radiative corrections (the Coleman-Weinberg potential) in the presence of matter was considered. Due to t-quark mass ($\sim 174~GeV$) a relatively high Higgs mass ($\sim 313~GeV$) was obtained. Only a fraction of this mass is connected to the Coleman-Weinberg potential ($m_{CW} \sim 15~GeV$). The model produces the first order phase transition for low temperature ($T_c \sim 10~GeV$). Formation of bubbles filled with matter was considered near the phase transition point. The realistic ball with $M \sim 10^5 - 10^9 M_\odot$ and the radius $R \sim 10^{12} - 10^{14}cm$ is obtained.

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1 Introduction.

The standard model describes the reality of the elementary particle interactions well known particularly in the perturbation sector. The standard model was built in analogy to the Landau-Ginsberg theory of superconductivity. It is natural to expect a phase transition [1] in similarity to superconductivity. If we believe, however that the electroweak theory is really the nonabelian one, we should also expect the nonperturbative effects in extreme conditions (sufficiently high temperature, high matter densities or gravitational force). One of the interesting features of the standard model is the scale invariance in the high symmetric phase. This is anomalous symmetry and quantum effects break it producing nonvanishing cosmological constant. The classical scale invariance joins the standard model to gravity. In this paper the electroweak theory will be extended by the dilatonic field and the singlet majoron field [2]. The dilaton field appears in a natural way in the Kaluza-Klein theories [3], superstring inspired theories [4], [5] and in the theories based on the noncommutative geometry approach [6]. The spontaneous global lepton symmetry breaking leads to appearance of the singlet majorana field [7] and the see-saw mechanism [8] of the neutrino mass generation.

In this paper it will be shown that due to the dilaton field interaction both standard model symmetry breaking scale and the global lepton symmetry breaking scales are connected to each other. It will be shown that the electroweak symmetry scale will be determined by the Coleman-Weinberg potential coming with the quantum corrections to the standard electroweak theory.

2 The theoretical background.

The Glashow-Weinberg-Salam dilatonic model with $SU_L(2) \times U_Y(1)$ symmetry is described by the Lagrange function

$$\mathcal{L} = \mathcal{L}_b + \mathcal{L}_f \tag{1}$$

$$
\begin{aligned}
\mathcal{L}_b &= -\frac{1}{4}e^{-2\kappa\varphi(x)}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}e^{-2\kappa\varphi(x)}B_{\mu\nu}B^{\mu\nu} \\
&+ \frac{1}{2}\partial_\mu\varphi(x)\partial^\mu\varphi(x) + (D_\mu H)^+ D^\mu H + \frac{1}{2}\partial_\mu\chi(x)\partial^\mu\chi(x) - U(H) - U(\chi)
\end{aligned}
\tag{2}
$$

with the $SU_L(2)$ field strength tensor

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon_{abc}W^b_\mu W^{c\nu} \tag{3}$$

and the $U_Y(1)$ field tensor

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{4}$$

The covariant derivative is given by

$$D_\mu = \partial_\mu - \frac{1}{2}igW^a_\mu \sigma^a - \frac{1}{2}g'YB_\mu \tag{5}$$

where B_μ and

$$W_\mu = \frac{1}{2}W^a_\mu \sigma^a \tag{6}$$

are a local gauge fields associated with the $U_Y(1)$ and $SU_L(2)$ symmetry group, respectively. Y is a hypercharge. The gauge group is simply the multiplication of $U_Y(1)$ and $SU_L(2)$ so there are two gauge couplings g and g'. Generators of the gauge groups are unit matrix for $U_Y(1)$ and Pauli matrixes for $SU_L(2)$. In the simplest version of the standard model a doublet of Higgs field is introduced

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{7}$$

with the Higgs potential

$$U(H) = \lambda(H^+H - \frac{1}{2}v_0^2 e^{-2\kappa\varphi(x)})^2 \tag{8}$$

The form of the potential leads to a degeneracy of the vacuum and to a nonvanishing vacuum expectation value of the Higgs field and in consequence to the fermion and boson masses. The similar type of potential we may expect for the complex field χ

$$U(\chi) = \lambda_S(\chi^+\chi - \frac{1}{2}u_0^2 e^{-2\kappa\varphi(x)})^2 \tag{9}$$

$U(\chi)$ has the global lepton U(1) symmetry.
The fermion contents of the model is extended only by the right handed neutrino ν_R as a singlet of the $SU_L(2) \times U(1)$ group. For simplicity, let us

limit ourselves to the first lepton family. We have the lepton lagrangian as follows:

$$\mathcal{L}_f = ie_R^+ \sigma^\mu \partial_\mu e_R + i\nu_R^+ \sigma^\mu \partial_\mu \nu_R + iL^+ \sigma^\mu D_\mu L + \tag{10}$$
$$ih_e(HLe_R + h.c) + ih_\nu(L^+ \epsilon H \nu_R + h.c.) + ih_R(\nu_R^2 \chi + h.c),$$

where $\sigma^\mu = \{I, \sigma^i\}$ Here we also adopted the notation

$$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, L = e, \nu, \tau \tag{11}$$

Let us consider a system of quantum boson fields

$$\phi_A = \{\varphi, W_\mu^a, B_\mu, H, \chi\} \tag{12}$$

$$\Phi_A = \sum_\lambda \frac{1}{\sqrt{(2\pi)^3}} \int \frac{d^3k}{\sqrt{2\omega_k}} \{a_{\mathbf{k},\lambda} e^{i\mathbf{k}\mathbf{x}-i\omega_k t} + a_{\mathbf{k},\lambda}^+ e^{-i\mathbf{k}\mathbf{x}+i\omega_k t}\} \tag{13}$$

$$[a_{\mathbf{k},\lambda}, a_{\mathbf{k'},\lambda'}^+] = g_{\lambda,\lambda'} \delta(\mathbf{k} - \mathbf{k'}) \tag{14}$$

with the vacuum state $|0>$ defined as $a_{\mathbf{k},\lambda}|0>= 0$ and a new system of quantum boson fields $\tilde{\phi}_A$ related to ϕ_A by

$$\phi_A = \tilde{\phi}_A + \xi_A , \tag{15}$$

where the shifts ξ_A are the classical fields. These shift transformations can be expressed as:
$$\tilde{\phi}_A = \mathcal{D}(\xi_A)\phi_A \mathcal{D}^+(\xi_A) ,$$

where
$$\mathcal{D}(\xi_A) = exp \sum_A \sum_\lambda \int d^3k (\xi_{A\mathbf{k}\lambda} a_{A\mathbf{k}\lambda}^+ - \xi_{A\mathbf{k}\lambda}^* a_{A\mathbf{k}\lambda}) .$$

Here $\sum_A$ is the sum over all shifted fields and $\sum_\lambda$ means the sum over all degrees of freedom for these fields. The $a_{A\mathbf{k}\lambda}$ and $a_{A\mathbf{k}\lambda}^+$ are the annihilation and creation operators for the ϕ_A field. The coefficients $\xi_{A\mathbf{k}\lambda}$ are the Fourier transformations of the ξ_A fields. Now we assume that in the Hilbert space $\mathcal{H}$ there exists a normalized vacuum vector $|0>$ which is annihilated by the operators $a_{A\mathbf{k}\lambda}$
$$a_{A\mathbf{k}\eta} \mid 0 >= 0 \ and \ < 0 \mid 0 >= 1 .$$

The shifts cause the changing of the ground state of a system according to the relation:

$$| 0 > \longrightarrow | \tilde{0} > = \mathcal{D}(\xi_A) | 0 > \ .$$

The new vacuum state $| \tilde{0} >$ is simply the Glauber coherent state. This state includes the infinite number of excited states of ϕ_A fields. The state $| \tilde{0} >$ is also normalized, i.e., $< \tilde{0} | \tilde{0} > = 1$. As the state $| 0 >$ is the vacuum state for the ϕ_A fields also the state $| \tilde{0} >$ may be considered as the vacuum state for the $\tilde{\phi}_A$ fields. Hence, when we have $< 0 | \phi_A | 0 > = 0$ we also have $< \tilde{0} | \tilde{\phi}_A | \tilde{0} > = 0$ and

$$< \tilde{0} | \phi_A | \tilde{0} > = < \tilde{0} | \tilde{\phi}_A | \tilde{0} > + \xi_A = \xi_A \ .$$

The point is that when the ground state $| \tilde{0} >$ is attained as the result of the transformation which is not the gauge symmetry transformation or as the result of the appearance of some new external charges in the system it leads to the conclusion that the Fock spaces which are built on the ground states $| 0 >$ or $| \tilde{0} >$, respectively, are not unitary equivalent. This means that some classical boson fields ξ_A may attain physical interpretation. The physical system is totally defined by the free energy

$$F = -kTlnTr(e^{-\beta H}) \tag{16}$$

where H is hamiltonian of the physical system

$$H = \sum_A \int d^3x \{ \partial_0 \Phi_A \pi_\Phi^A - \mathcal{L} \} \tag{17}$$

and $\pi^A = \frac{\partial \mathcal{L}}{\partial(\partial_0 \Phi_A)}$ is a momentum connected to Φ_A. In this paper we shall use the effective potential approach built using the Bogolubov inequality [9]

$$F \leq F_1 = F_0(m^2) + < H - H_0 >_0 \tag{18}$$

F_0 is the free energy of the trial system

$$F_0 = U_{CW} + \sum_A \{ \frac{1}{24} m_A^2 T^2 - \frac{1}{12\pi} m_A^3 T - \frac{m_A^4}{64\pi^2} ln(\frac{m_A^2}{cT^2}) \} + ... \tag{19}$$

with $c = \frac{3}{2} + 2ln(4\pi) - 2\gamma \cong 5.4$. U_{CW} is the Coleman-Weinberg potential. The hamiltonian of the system is defined as usual as

$$H = \sum_A \int d^3x (\pi_{\Phi_A} \dot{\Phi}^A - \mathcal{L}) \tag{20}$$

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The trial system we shall suppose as effectively free quasiparticle system described by the Lagrange function

$$\mathcal{L}_0 = \sum_A \frac{1}{2} \partial_\mu \tilde{\Phi}_A \partial^\mu \tilde{\Phi}_A - \frac{1}{2} m_A^2 \tilde{\Phi}_A^{\ 2} \tag{21}$$

We decompose the Φ_A field into two components, the effectively free quasiparticle field $\tilde{\Phi}_A$ and the classical boson condensate ξ_A

$$\Phi_A = \tilde{\Phi}_A + \xi_A \tag{22}$$

The ξ_A field will be treated as the variational parameters in the effective potential.

3   The electroweak phase transition.

The standard model was built in analogy to the Landau-Ginsberg theory of superconductivity where is the continuous phase transition. Indeed, in the first approximation we have only condensation of the Higgs field.

$$H = \left(\begin{array}{c} H^+ \\ \frac{1}{\sqrt{2}} v \end{array} \right) = \tilde{H} + \left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}} v_0 \end{array} \right) \tag{23}$$

This happens if we neglect the radiative corrections giving the Coleman-Weinberg potential. Including only temperature effects the minimal standard electroweak model has the effective potential

$$U_{eff} = \frac{1}{2} D T^2 v^2 + \frac{1}{4} \lambda (v^2 - v_0^2)^2 \tag{24}$$

with

$$D = \sum_A \frac{1}{12} \left(\frac{m_A}{v_0} \right)^2 = \frac{1}{12} \{ \left(\frac{m_W}{v_0} \right)^2 + \left(\frac{m_Z}{v_0} \right)^2 + \left(\frac{m_H}{v_0} \right)^2 \} \tag{25}$$

This potential has temperature dependent minimum

$$v_T^2 = v_0^2 - \left(\frac{D}{\lambda} \right) T^2 \tag{26}$$

which vanishes at phase transition point

$$T_c = \sqrt{\frac{D}{\lambda}} v_0 = \frac{1}{2D} m_H \tag{27}$$

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This phase transition temperature is really high, for example for $m_H = 302\ GeV$ we have $T_c = 464.6\ GeV$.

In the extended standard model we may expect condensation of the following fields

$$\varphi = \tilde{\varphi} + \sigma \qquad or \qquad D = e^{-\kappa\sigma} \tag{28}$$

$$W_\mu^a = \tilde{W}_\mu^a + a_\mu^a \tag{29}$$

$$B_\mu = \tilde{B}_\mu + b_\mu \tag{30}$$

$$H = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}v \end{pmatrix} = \tilde{H} + \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v_0 \end{pmatrix} \tag{31}$$

$$\chi = \tilde{\chi} + \frac{1}{\sqrt{2}}u_0 = \frac{1}{\sqrt{2}}(u + i\varphi_M) \tag{32}$$

with $\tilde{\chi} = \frac{1}{\sqrt{2}}(\varphi_u + i\varphi_M)$. φ_M is the majoron field. In the result of bosons condensation the Higgs mechanism generates not only the Dirac mass

$$m_{D\nu} = \frac{1}{\sqrt{2}}h_e v_0 \tag{33}$$

but the lepton number violating Majorana mass

$$M = \frac{1}{\sqrt{2}}h_\nu u_0 \tag{34}$$

as well. Thus the neutrino mass matrix can be written as follows

$$\mathcal{M} = \begin{pmatrix} 0 & m_{D\nu} \\ m_{D\nu} & M \end{pmatrix}$$

In the case $M = 0$ only the Dirac neutrino may be obtained. In general, it should have the same mass as the electron or quark ($\sim 1\ MeV$). In the broken phase due to the see-saw mechanism we obtain two Majorana mass eigenstates [10]

$$m_{\nu,M} = \frac{1}{2}M\{1 \mp \sqrt{1 + 4(\frac{m_{D\nu}}{M})^2}\} \sim \{-\frac{m_{\nu,D}^2}{M}, M\} \tag{35}$$

The astrophysical boundaries [11] suggest that $u_0 \sim v_0$ and that Yukawa coupling $h_\nu = 10^{-17}$ is small. This gives the mass of the Dirac neutrinos $m_{\nu,D} \sim 25KeV$ If we estimate $M \sim 100$ GeV we have $m_{1,\nu,M} \sim 2.7\ 10^{-3}$ eV

and $m_{2,\nu,M} \sim 100$ GeV for the Majorana neutrinos. The classical potential U gives

$$U_0(v,\sigma) = \frac{1}{4}\lambda(v^2 - v_0^2 e^{-2\kappa\varphi(x)})^2 + \frac{1}{4}\lambda_S(u^2 - u_0^2 e^{-2\kappa\varphi(x)})^2 \tag{36}$$

$$= \frac{1}{4}\lambda(v^2 - v_0^2 D^2)^2 + \frac{1}{4}\lambda_S(u^2 - u_0^2 D^2)^2$$

$$\frac{\partial U_0}{\partial v} = 0 \quad gives \quad \lambda(v^2 - v_0^2 D^2)v = 0 \tag{37}$$

$$\frac{\partial U_0}{\partial u} = 0 \quad gives \quad \lambda(u^2 - u_0^2 D^2)u = 0 \tag{38}$$

$$\frac{\partial U_0}{\partial D} = 0 \quad gives \quad -\lambda(v^2 - v_0^2 D^2)v_0^2 D - \lambda_S(u^2 - u_0^2 D^2)u_0^2 D = 0 \tag{39}$$

Apart of the trivial solution $v = 0$, $D = 0$ we have

$$D^2 = \frac{v^2}{v_0^2} \tag{40}$$

and

$$u^2 = u_0^2 D^2 = \frac{u_0^2}{v_0^2}v^2 \tag{41}$$

It is interesting that in the presence of the dilaton $\varphi(x)$ the classical potential vanishes at the minimum point

$$U_0(v) = U_0(v, u = (u_0/v_0)v, D = v/v_0) = 0 \tag{42}$$

There is no cosmological term on the classical level. Now we can define the standard model Higgs field φ_v, Higgs field φ_u connected to the global $U_L(1)$ symmetry and dilaton field φ_d as

$$v = v_0 + \varphi_v \tag{43}$$

$$u = u_0 + \varphi_u \tag{44}$$

$$d = v_0 + \kappa v_0 \varphi_d \tag{45}$$

The Higgs field mass is determined from the "mass matrix"

$$m_{i,j}^2|_{min} = \frac{\partial^2 U_0}{\partial \Phi_i \partial \Phi_j} \tag{46}$$

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$$m_{i,j}^2|_{min} = \begin{pmatrix} 2\lambda v_0^2, & 0, & -2\lambda v_0^2(\kappa v_0) \\ 0, & 2\lambda_S u_0^2, & -2\lambda_s u_0^2(\kappa v_0) \\ -2\lambda v_0^2(\kappa v_0), & -2\lambda_s u_0^2(\kappa v_0), & 2(\kappa v_0)^2(\lambda v_0^2 + \lambda_S(\frac{u_0}{v_0})^2 u_0^2) \end{pmatrix} \quad (47)$$

where now the classical fields are $\Phi = (\varphi_v, d = \varphi_v)$. At the extremum point the diagonalized mass matrix has the form

$$diag \ m_{i,j}^2|_{min} = \ \{m_H^2 = 2\lambda v^2, m_L^2 = \lambda_S u_0^2, m_d = 0\} \quad (48)$$

The physical fields are result of diagonalization of this mass matrix. They may be defined as $\Phi_{i,ph} = (\varphi_H, \varphi_L, \varphi_D)$ The physical fields are an orthogonal mixture $\Phi_{i,ph} = (\varphi_u, \varphi_u, \varphi_d)$

$$\Phi_{i,ph} = R_i^j \Phi_j \quad (49)$$

where R_i^j is orthogonal matrix diagonalizing the mass matrix (47). As $(\kappa v_0) \sim 10^{-17}$ is really very small number, this mixing is very small. $\varphi_H \sim \varphi_v$ is a standard model Higgs particle, $\varphi_L \sim \varphi_u$ is a Higgs particle connected to the spontaneous $U_L(1)$ symmetry breaking, φ_D is the dilaton field. We have not determined on the classical level. It must be determined by the radiative corrections — the Coleman-Weinberg effective potential [12]. The gauge field condensation

$$a_\mu^a = \{a_0^a = \zeta \delta_0^a, a_i^a = 0\} \quad (50)$$

$$b_\mu = \{b_0 = \eta, b_i = 0\} \quad (51)$$

For example we have

$$D_\mu H^+ D^\mu H = \frac{1}{4}g^2 v^2 \zeta^2 + \frac{1}{4}g'^2 v^2 \eta^2 + \frac{1}{2}M_Z^2 Z_\mu Z^\mu + \frac{1}{8}g^2 v^2 \frac{g'}{\sqrt{g^2 + g'^2}} a_0^3 Z^0 \ + ... \quad (52)$$

We have used redefinition of the gauge field

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} cos\vartheta_W & sin\vartheta_W \\ -sin\vartheta_W & cos\vartheta_W \end{pmatrix} \begin{pmatrix} Z_\mu^3 \\ A_\mu \end{pmatrix}$$

with

$$cos\vartheta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

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the W and Z bosons masses are the same as in the standard model

$$M_W^2 = \frac{1}{4}g^2 v^2$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$$

$$Z_\mu = \tilde{Z}_\mu + z_\mu$$

$$A_\mu = \tilde{A}_\mu + a_\mu$$

where

$$a_0 = cos\vartheta_W \zeta - sin\vartheta_W \eta$$

If we do not want to break the $U_Q(1)$ electromagnetic gauge symmetry, we should impose the condition

$$z_0 = \frac{\eta}{cos\vartheta_W} \tag{53}$$

This gives

$$\mathcal{L} = \frac{1}{2}M_Z^2(v)z_0^2 + \rho_Z z_0 \ + \ ... \tag{54}$$

where

$$\rho_Z \Leftarrow \mathcal{J}_Z^\mu = \mathcal{J}_W^{3,\mu}cos\vartheta_W + \mathcal{J}_Y^\mu sin\vartheta_W + \tag{55}$$

$$\frac{\partial \mathcal{L}}{\partial z_0} = 0 \quad gives \quad z_0 = -\frac{\rho_Z}{M_Z^2(v)} \tag{56}$$

In the presence of the weak external neutral charge ρ_Z we have an additional term

$$U_{add} = \frac{1}{2}\frac{\rho_Z^2}{M_Z^2(v)} \tag{57}$$

$$U(v) = U_{add}(v) + U_{CW}(v) \tag{58}$$

where the quantum corrections ($\sim \hbar$) generate the Coleman - Weinberg potential Keeping only the contributions associated with the gauge bosons W, Z and the quark t the radiative corrections give

$$U_{CW}(v) = \sum_{i=\varphi_H,W,Z,t,...} \frac{n_i}{64\pi^2}m_i^4(v)\{ln\frac{m_i^4(v)}{Q^2} - \frac{3}{2}\} \tag{59}$$

where for example

$$m_W^2 = \frac{1}{4}g^2 v^2, \tag{60}$$

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2, \tag{61}$$

$$m_t^2 = h_t^2 v^2, ... \tag{62}$$

n_i depends on the number of degrees of freedom and the particle's statistics

$$n_{\varphi_H} = 1, \ n_W = 6, \ n_Z = 3, \ n_t = -12, ... \tag{63}$$

Q is the renormalization scale. Let us notice that $V_r(0) = 0$. In our calculation we should also include the contribution from the $U_L(1)$ Higgs field and from the right handed neutrino ν_R. They have masses

$$m_L^2 \sim 2\lambda_S (\frac{u_0}{v_0})^2 v^2 \tag{64}$$

$$m_{\nu_R}^2 = h_{\nu_R}^2 (\frac{u_0}{v_0})^2 v^2 \tag{65}$$

Their number of degrees of freedom are

$$n_L = 1, n_{\nu_R} = 2*3 \tag{66}$$

where 2 is the spin degree of freedom and 3 is the number of fermion families. The Coleman-Weinberg potential may be written in the form

$$U_{CW}(x) = \frac{1}{4}Cv^4\{ln\frac{v^2}{Q^2} - \frac{25}{6}\} \tag{67}$$

where

$$C = \frac{1}{16\pi^2}\{6(\frac{m_W}{v_0})^4 + 3(\frac{m_Z}{v_0})^4 + (\frac{m_H}{v_0})^4 + (\frac{m_L}{v_0})^4 - 12(\frac{m_t}{v_0})^4 - 6(\frac{m_{\nu_R}}{v_0})^4\} \tag{68}$$

The minimum point v_0 define

$$v_0 = Qe^{\frac{11}{6}} \tag{69}$$

Finally we have

$$U_{CW} = \frac{1}{4}Cv^4\{ln(\frac{v^2}{v_0^2}) - \frac{1}{2}\} \tag{70}$$

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It is interesting that the radiative corrections produce a negative cosmological constant

$$B = -\frac{1}{128}\{6m_W^4 + 3m_Z^4 + m_H^4 + m_L^4 - 12m_t^4 - 2m_{\nu_R}^4\} \tag{71}$$

This puts the cosmological boundaries on the quark top mass. For example for $M_t \sim 174 \; GeV$ [13] $m_{\nu_R} = 120 \; GeV$ and $m_L = 250 \; GeV$ we have $m_H > 104 \; GeV$ and $B = 10^5 \; GeV^4$. When we include the Coleman-Weinberg potential the mass matrix will change a bit

$$m_{i,j}^2|_{min} = \frac{\partial^2 U_0}{\partial \Phi_i \partial \Phi_j} \tag{72}$$

$$m_{i,j}^2|_{min} = \begin{pmatrix} 2\lambda v_0^2 + \frac{\partial^2 U_{CW}}{\partial v^2}, & 0, & -2\lambda v_0^2(\kappa v_0) \\ 0, & 2\lambda_S u_0^2, & -2\lambda_s u_0^2(\kappa v_0) \\ -2\lambda v_0^2(\kappa v_0), & -2\lambda_s u_0^2(\kappa v_0), & 2(\kappa v_0)^2(\lambda v_0^2 + \lambda_S(\frac{u_0}{v_0})^2 u_0^2) \end{pmatrix} \tag{73}$$

For small $(\kappa v_0) \sim 10^{-17}$ the Higgs particle mass is equal to

$$m_H^2 = m_0^2 + m_{CW}^2 \tag{74}$$

with tree level Higgs mass

$$m_0^2 = 2\lambda v_0^2 \tag{75}$$

and the Coleman-Weiberg Higgs mass

$$m_{CW}^2 = \frac{\partial^2 U_{CW}}{\partial v^2} = 2Cv_0^2 \tag{76}$$

The dilaton mass is equal to

$$m_D^2 = \frac{1}{2}m_0^2(\kappa v_0)^2(1 - \frac{m_{CW}^2}{m_0^2 + m_{CW}^2}) \quad \sim \quad 10^{-6} \; eV \tag{77}$$

Temperature contributions to the effective potential originated from F_0 may also be included. At last the effective potential has the form

$$U_{eff} = \frac{1}{2}DT^2v^2 + \frac{1}{4}Cv^4\{ln(\frac{v^2}{v_0^2}) - \frac{1}{2}\} + U_{add} \tag{78}$$

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with

$$D = \sum_A \frac{1}{12}(\frac{m_A}{v_0})^2 = \frac{1}{12}\{(\frac{m_W}{v_0})^2 + ... \tag{79}$$

Let us neglect for the moment U_{add}. The extremum U_{eff} points v_T obey the equation

$$DT^2 + Cv_T^2 ln(\frac{v_T^2}{v_0^2}) = 0 \tag{80}$$

At the first order phase transition point T_c we have the degenerate values of the free energy (effective potential). This means that

$$U_{eff}(0) = U_{eff}(v_c) \tag{81}$$

with $v_c = v_{T_c}$. This condition defines the first order phase transition temperature T_c

$$T_c^2 = \frac{C}{2D}v_c^2 \sim \frac{C}{2D}v_0^2 \tag{82}$$

The known particles masses allows us to establish boundaries on Higgs particle mass and quark top mass [14] $(c > 0)$ and the phase transition temperature. For example in the minimal standard model (without dilaton and majoron field) we have $m_t < 100\ GeV$. For two Higgs doublet like in the supersymmetrical extension we have $m_t < 100\ GeV$. In the model built on noncommutative geometry there is limitation $m_t \sim 130 GeV$. In the extended model if $m_t > 89\ GeV$ then Majorana neutrino mass $m_i < 130 GeV$. All these estimations allows us to predict the phase transition temperature [Table 1]

$$T_c \sim 10\text{--}30\ GeV \tag{83}$$

It is rather low temperature in comparison to the minimal standard model $(T_C = 464.6 GeV$ for $m_H = 302 GeV)$.

4 The astrophysical meaning.

Let us consider now the nonhomogeneous Higgs field configuration [15] near the first order phase transition point. In the presence of fermions we shall have two (v_*, v) different from zero minima (Fig. 1). These two minima are

degenerate at the phase transition point $T_c \sim 10 - 20 GeV$. When $T_c \to 0$ we have the limit $(v_* = 0, v = v_0)$. Let us now define the effective field

$$\Phi = \frac{1}{\sqrt{2}}(v - v_*) = \frac{1}{\sqrt{2}}x \tag{84}$$

Effectively, the Higgs field may be described near the phase transition point as

$$\mathcal{L} = \partial_\mu \Phi^+ \partial^\mu \Phi - U(\Phi) \tag{85}$$

with

$$U(\Phi) = \lambda_*(\Phi^+\Phi)|\Phi - \frac{1}{\sqrt{2}}x_0|^2 \tag{86}$$

The parameter λ_* determines the potential wall height between two minima. In the first approximation

$$U_0 = \frac{1}{16}\lambda_* v_0^4 \tag{87}$$

The Lagrange equation gives

$$\Box x = \frac{\partial U}{\partial x} \tag{88}$$

In the spherical coordinates this equation takes the form

$$\frac{d^2x}{dr^2} + \frac{2}{r}\frac{dx}{dr} = \frac{\partial U}{\partial x} \tag{89}$$

As the potential takes the degenerate form

$$U = \lambda' x^2 (x - x_0)^2 \tag{90}$$

where x_0 is obtained from the Coleman-Weinberg potential U_{CW}. In the thin wall approximation we neglect the second term in equation (89). As a result we obtain a one dimensional equation which is easy to solve.

$$\frac{1}{2}(\frac{dx}{dr})^2 = U \tag{91}$$

In this approximation the solution may be described as the ball with the radius R

$$x = \left\{ \begin{array}{cc} 0 & r \leq R \\ x_0 e^{m_\Phi(r-R)}/(1 + e^{m_\Phi(r-R)}) & r > R \end{array} \right\} \tag{92}$$

where

$$m_\Phi^2 = \frac{\partial U_{ef}}{dx}|_{x_0} \tag{93}$$

defines the scalar Higgs field mass. Its inverse $l = 1/m$ is the coherent length and measures the ball wall size. So, the wall is really thin and this approximation seems to be reasonable. The scalar ball may be thought of as a constant solution inside the ball and the soliton solution representing the wall. From the solution (92) one can conclude that inside the ball for $r < R$ $x = 0$. That means that inside the ball exists a phase with the gauge boson (Z_μ) condensation. Outside the soliton $(x \neq 0)$ we have the low symmetry phase with the broken electroweak symmetry. In this region all fermions get masses. Because inside the ball all fermions are nearly massless whereas outside they get large masses, they have the natural tendency to fill the ball. They will give the stabilizing (repulsive) term in the expression for the total energy of the whole system which will protect from the gravitational collapse. The boson part of the ball energy

$$\mathcal{E}_b = \frac{4\pi}{3}B + 4\pi \int_R^\infty dr r^2 \{\frac{1}{2}(\frac{dx}{dr})^2 + U_{ef}(x)\} \tag{94}$$

In the thin wall approximation we have

$$\mathcal{E}_b = sR^2 + \frac{4\pi}{3}BR^3 \tag{95}$$

where

$$s \sim \frac{\sqrt{2\lambda_*}}{8}x_0^3 \tag{96}$$

is the surface tension. The fermion energy corresponds to the repulsive force coming from the Pauli principle

$$\mathcal{E}_f = \frac{AN^{\frac{4}{3}}}{R} \tag{97}$$

with

$$A = \frac{4}{3}\pi^2(\frac{9\pi}{2})^{\frac{4}{3}}\gamma^{-\frac{1}{3}} \tag{98}$$

where γ is a number of degrees of freedom. The total energy of the ball is equal

$$\mathcal{E} = \frac{N^{\frac{4}{3}}A}{R} + sR^2 + \frac{4\pi}{3}BR^3 \tag{99}$$

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In this model we consider the case when the bag constant $B = 0$ so in the expression for total energy remains only the term containing the surface tension s.

$$\mathcal{E} = \frac{N^{\frac{4}{3}}A}{R} + sR^2 \qquad (100)$$

For example, for $x_0 \sim v_0 = 246 GeV$ $s = 2.7 \ 10^5 (GeV)^3$. Minimizing, $\frac{\partial \mathcal{E}}{\partial R} = 0$ gives

$$R_0 = (\frac{A}{2s})^{\frac{1}{3}} N^{\frac{4}{9}} \qquad (101)$$

and

$$\mathcal{E}_0 = \frac{3A}{2R_0} N^{\frac{4}{3}} \qquad (102)$$

Because inside the ball all fermions are massless in the first approximation $v_* \sim 0$ whereas outside they get large masses, they have the natural tendency to fill the ball. They will give the stabilizing (repulsive) term in the expression for the total energy of the whole system which will protect from the gravitational collapse. This picture will be energetically favourable until the Fermi level ε_F doesn't exceed the value of the fermion masses in the broken symmetry phase. It means that inside the ball fermions whose masses outside the ball are larger then inside (t-quark etc) dominate. Because inside the ball energy of the supersymmetric ground state equals zero whereas outside the ball the cosmological constant (energy of the ground state) also equals zero, the potential $U(x)$ describes a soliton solution with the bag constant $B = 0$ and only with different from zero surface tension. We can notice a similarity to a quark star. In this case there is a deconfinement phase inside the soliton with $B \neq 0$ which is decisive for macroscopic properties of the star. In our case $B = 0$ and the values of the bag mass and radius are determined only by the surface tension. We shall have the critical radius R_c when $R_g = R_0$. As $R_g = \frac{2\mathcal{E}}{M_{Pl}^2}$, we have the critical fermion number N_c defined as

$$N_c = \frac{1}{A} \left(\frac{M_{Pl}^2}{3\sqrt{4s^2}} \right)^3 \qquad (103)$$

$$R_c = \sqrt{\frac{3A}{M_{Pl}}}, \qquad \mathcal{M}_c = \frac{1}{2} M_{Pl}^2 R_c$$

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The previous numerical parameters give $N_c = 5.27 \times 10^{69}$, $R_c = 2.37 \times 10^{14}$ cm and $M_c = 8.06 \times 10^9 \ M$. The ball mass will depend as $N^{\frac{8}{9}}$, while the energy of the corresponding N free particles will depend linearly on N. This suggests that the ball is stable considering the decay of free particles. The proposed gauge field condensation was the modest one. More sophisticated condensation including W_μ bosons may be also considered [16]. What is very interesting and a bit anxious is that it breaks the electric charge conservation. Fortunately, this phase is energetically unstable. The current idea of a quasar is that its energy comes from the matter accretion on the supermassive black holes. Nevertheless this model can not solve many astrophysical problems associated for example with the early formation of such massive black holes [17]. An alternative explanation of a quasar is connected with the phenomena of phase transitions in the early universe. The grand unification theory predicts the sequence of phase transitions during the evolution of the early universe. If they are discontinued then the bubbles of the new low temperature phase will appear during the universe expansion. After the phase transition point the low temperature phase will dominate and the areas of the old high temperature phase also will form bubbles. In the presence of fermions inside, the soliton is stabilized by surface tension term ($\sim R^2$). As the result the equilibrium configuration appears with definite mass and radius. The comparatively late phase transition takes place in the standard model during the spontaneous symmetry breaking from $SU_L(2) \times U_Y(1)$ to $U_Q(1)$. If such a phase transition is discontinued then the bubbles of the high temperature phase filled for example with neutrinos may be produced. In this paper it was shown that inside the bubble we have only the Dirac neutrino with mass of the order of the electron or quark mass. This implies that the total lepton number is conserved inside the ball. In the broken phase two Majorana mass eigenstates were obtained. If we put such bubbles into the interstellar medium they may produce the identical accretion as we expect from the supermassive black holes. According to Holdom [18],[19],[20] a lifetime to the value of Fermi level in the ball. For a ball with a lifetime comparable to the age of the Universe gravitational interactions would prevail and the conversion of mass could be compared with that in the black hole model.

The existence of balls can be connected with a cosmological phase transition in the Standard Model extended to the case with the lepton symmetry breaking. Balls are created in the early Universe as a consequence of a quan-

tum tunnelling effect or by thermal fluctuations. Empty balls tend to shrink and disappear. In order to stabilize them in the highsymmetric phase Dirac neutrinos are present. However, after the phase transition two neutrinos appear as the result of the see-saw mechanism. One of them possesses a very big mass whereas the mass of the second neutrino is small. It is natural for heavy neutrinos to fall to the interior of the ball where they are almost massless. This is the same mechanism that leads to the quark confinement in the Friedberg-Lee nontopological model of hadrons. Fermions falling into the ball stabilize it and cause the increase of the ball radius and mass.

The number of bubbles which survive and their sizes depend mainly on the fermion density in the early Universe. Perhaps it would be relevant to look for any correlations with the dark matter. These balls could be good candidates for compact dark objects.

5 Conclusion

In this paper we have shown the possibility of boson condensation in the extreme conditions in the standard electroweak theory. The first order phase transition induced by the radiative corrections (the Coleman-Weiberg potential) in the presence of matter was considered. Due to t-quark mass ($\sim 174 \ GeV$) a relatively high Higgs mass ($\sim 313 \ GeV$) was obtained. Only a fraction of this mass is connected to the Coleman-Weinberg potential ($m_{CW} \sim 15 \ GeV$). The model produces the first order phase transition for low temperature ($T_c \sim 10 \ GeV$). Such a phase transition may have an astrophysical meaning, and may be connected to the baryogenesis for the electroweak scale [21]. The realistic ball with $M \sim 10^5 - 10^9 M_\odot$ and the radius $R \sim 10^{12} - 10^{14} cm$ is obtained.

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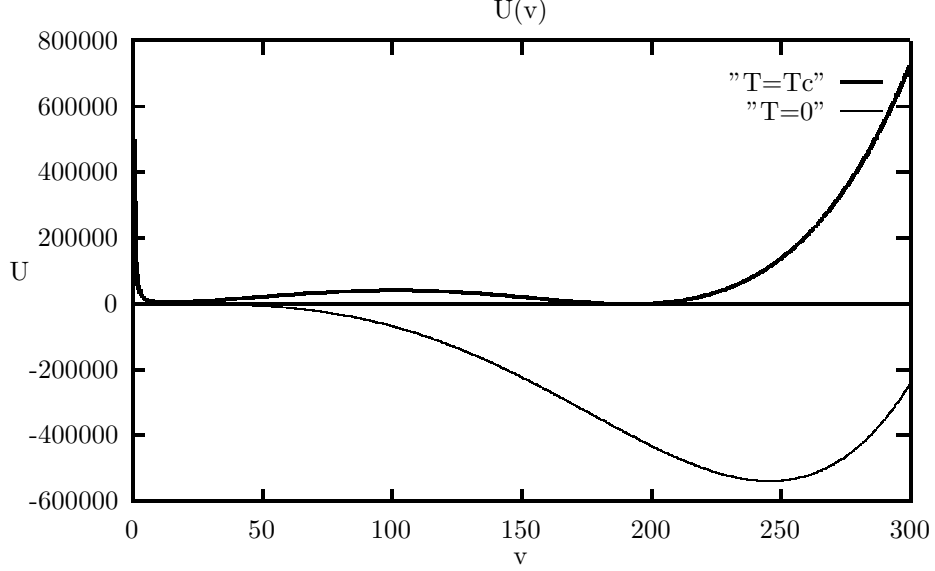
Acnowledgments

Figure 1. The potential $U(x)$ for external fermions densities $\rho_Z \neq 0$ for temperature $T = 0$ and $T = T_c$.

Table 1: The value of cosmological constant B, phase transition temperature T_c and Higgs total mass m_H for different parameters of the Coleman-Weinberg Higss mass m_{cw}

B	T_c	m_H	m_{CW}
$-9.2\ 10^5\ (GeV)^4$	$10.0\ GeV$	$313.0\ GeV$	$15\ GeV$
$-2.5\ 10^6\ (GeV)^4$	$15.9\ GeV$	$327.7\ GeV$	$25.9\ GeV$
$-5.6\ 10^6\ (GeV)^4$	$22.9\ GeV$	$352.7\ GeV$	$37.2\ GeV$